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IMPROVED VERSION OF THE EIKONAL MODEL FOR ABSORBING SPHERICAL PARTICLES

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Abstract

We present a new expression of the scattering amplitude, valid for spherical absorbing objects, which leads to an improved version of the eikonal method outside the diffraction region. Limitations of this method are discussed and numerical results are presented and compared successfully with the Mie theory.

Key-Words : Light scattering by spheres, eikonal approximation.

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1 Introduction

The eikonal picture, or eikonal model, has been first developped by Perrin and Chiappetta [1] to describe optical scattering phenomenons by scatterers having an axis of symmetry parallel to the incident wave-vector. This approximation is a variant of the High-Energy Approximation (HEA) introduced by Glauber [2] for scattering of particles by a potential of infinite range. Even though this latter theory is valid only for small scattering angle, the eikonal picture was proven to be valid in the whole angular domain. Numerical results [1]-[3] for the sphere shown rather good agreement with the Mie theory, particularly when internal multiple reflexions are negligible. This useful approximation provides a simple mathematical scheme and has been used particularly in astronomy, to describe the light scattering by irregular particles [4].

An other version of the eikonal method, called the generalized eikonal approximation [5], including the second Born correction of the scattering amplitude has been developped for the sphere, leading to good agreement with the Mie theory. Recently, a vectorial version of the eikonal method valid for objects of approximately spherical shape has been derived [6]. The eikonal approximation has also been extended to the case of objects whose axis of symmetry is not parallel to the direction of the incident wave-vector [7].

The eikonal picture applied to scattering by large spheres gives only a qualitative description of the backward intensity. In the present work, we show that simple considerations suggest a modification of the eikonal model amplitude for large absorbing spheres, leading to a better quantitative description of the scattering phenomena.

This paper is organized as follows : in section 2, we start by recalling the eikonal model and we give the scattering amplitude for an absorbing sphere. In section 3, we consider this scattering amplitude in the forward and backward limits. Comparisons with the diffraction and the optical backscattering amplitudes suggest an empirical transformation of the scattering amplitude which is tested against exact solutions in section 4. The conclusion is given in section 5.

2 The eikonal scattering amplitude for an absorbing sphere.

The eikonal method leads to an approximate formulation of the solution of the scalar Helmholtz equation

$$\Big\{\Delta + m^2k^2\Big\}\Psi(r) = 0, \tag{1}$$

where m is the refractive index of the scatterer, $\mathbf{k}$ is the wave-vector of the incident plane wave, and $\Psi(r)$ is the electric field perpendicular to the scattering plane [3]. For objects having an axis of symmetry parallel to $\mathbf{k}$ the solution $\Psi(r)$ can be written for large r :

$$\Psi(r) = e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta)\frac{e^{ikr}}{r} + O(\frac{1}{r^2}). \tag{2}$$

The scattering amplitude is given [1], in cylindrical coordinates, by :

$$f(\theta) = -\frac{k^2}{2}\int_0^\infty dbbJ_0(kb\sin\theta)G(b) + O(1), \tag{3}$$

where $G(b)$ is

$$G(b) = \int_{-\infty}^{\infty} dzU(b,z)e^{-iqz}\exp\Big\{i\frac{k}{2}\int_{-\infty}^{z(b)} dz'U(b,z')\Big\}, \tag{4}$$

$z(b)$ is the boundary of the scatterer for a given value of the impact parameter b, and $U(b,z) = 1 - m^2$, $q = 2k\sin^2\frac{\theta}{2}$. The incident wave-vector $\mathbf{k}$ has been taken parallel to the z axis (see Fig. 1). The function $G(b)$ can be computed exactly when the refractive index is a constant. For a convex particle whose center of symmetry is located at the origin :

$$G(b) = \frac{2}{k}\gamma(\theta)\Big\{e^{-iqz(b)+i\chi(b)} - e^{iqz(b)}\Big\}, \tag{5}$$

with $\gamma(\theta) = \dfrac{kU}{2q - kU}$ and the eikonal function $\chi(b)$ reads :

$$\chi(b) = \frac{k}{2}\int_{-\infty}^{z(b)} dz'U(b,z'). \tag{6}$$

For a sphere of radius a, $\chi(b) = kUz(b)$. Then the scattering amplitude has the simple expression :

$$f(\theta) \approx -ik\gamma(\theta) \int_0^a dbbJ_0(kb\sin\theta)\Big\{e^{-iqz(b)+i\chi(b)} - e^{iqz(b)}\Big\}, \tag{7}$$

with $z(b) = \sqrt{a^2 - b^2}$.
When the refractive index has a non negligible imaginary part, *i.e.* when the condition $|Im(m)|ka \gtrsim 1$ is satisfied, the real part of $i\chi(b)$ takes a high negative value which leads to the approximate expression of $f(\theta)$, namely :

$$f(\theta) \approx ik\gamma(\theta) \int_0^a dbbJ_0(kb\sin\theta)e^{iqz(b)}. \tag{8}$$

3 Modification of the eikonal model

Assuming that the sphere is sufficiently absorbing means that physically, the diffractive and the first order reflective parts of the scattering amplitude give a reasonable description of the scattering phenomenon. The present expression (7) of the scattering amplitude leads only to qualitative results in the backward direction and needs to be modified. Let us first consider the behaviour of $f(\theta)$ in (8) when θ is small (*i.e.* when the diffractive part dominates). Since, $e^{iqz(b)} \approx 1$, we obtain :

$$f(\theta) \approx \frac{ia}{\sin\theta} J_1(ka\sin\theta), \tag{9}$$

showing that we recover the exact expression of the diffraction amplitude for a sphere. On the other hand, in the neighbourhood of the backscattering angle (*i.e.* $|\pi - \theta| << 1$), the use of the method of stationary phase leads to :

$$f(\theta) \approx \frac{a}{2\sin^2\frac{\theta}{2}} \exp{(2ika\sin^2\frac{\theta}{2})}\gamma(\theta). \tag{10}$$

Since $\sin^2\frac{\theta}{2}$ is close to 1, we approximate (10) as :

$$f(\theta) \approx \frac{a}{2} \exp{(2ika\sin\frac{\theta}{2})}\gamma(\theta). \tag{11}$$

Whereas the geometrical optics gives :

$$f_{Opt} \approx \frac{a}{2} \exp{(2ika\sin\frac{\theta}{2})}r_\perp(\theta), \tag{12}$$

where $r_{\perp}(\theta)$ [8] is the Fresnel reflection coefficient for the perpendicular component of the electric field, which was proven in [3] and [6] to be identical to the eikonal amplitude. We observe that the expression (11) differs from (12) by a factor $\frac{\gamma(\theta)}{r_{\perp}(\theta)}$ which has a modulus different from 1 for refractive index near 1, leading for large angle to a systematic error in the evaluation of the scattering intensity. However, since $r_{\perp}(0) = \gamma(0)$, a simple modification of the eikonal model which takes into account the forward and the backward asymptotic behaviors (9) and (12), would consist in replacing $\gamma(\theta)$ by $r_{\perp}(\theta)$, leading to :

$$f_{mod}(\theta) \approx ikr_{\perp}(\theta) \int_0^a dbbJ_0(kb\sin\theta)e^{iqz(b)}. \tag{13}$$

This transformation has the advantage of being able to reproduce the two main components of the scattering amplitude valid respectively in the forward and the backward scattering directions. The comparison between the modified scattering intensity and the Mie theory, will be given in the next section.

4 Numerical results

In this section, we will compute the scattering intensity $i(\theta) = k^2|f(\theta)|^2$ for homogeneous, large spheres ($ka \geq 20$), from the original eikonal model (7) and the modified one (13). The model is in fact valid for $ka \geq 10$, and according to a numerical point of view, the new version is strictly equivalent to the original one when the expression (8) is used. The figures (2-6) show the results obtained when the refractive index has an imaginary part sufficiently large (according to the relation written above). We observe that the modified eikonal formula (dashed curves) is in much better agreement with the Mie theory (solid curves) for scattering angles greater than 40^o, as long as $|Im(m)|ka \gtrsim 1$ and $ka >> 1$. Indeed, in this angular region, since the reflection becomes the main component of the scattering amplitude, the intensity predicted by the original eikonal model (dotted curves) lies systematically above the Mie theory. We observe that no oscillations appear for angles more or less greater than 40^o because the internal multiple reflections in the sphere are not included in the new formulation, leading to a smooth description of the scattered intensity clearly seen in Fig. 6. In Fig. 7, we

have considered the case of an ellipsoid with $ka = 50$ and $kb = 75$ in the z direction. Although no comparison can be made with respect to the Mie theory, there is again a difference between the modified eikonal model (dashed curve) and the original one (dotted curve) for scattering angles above 40^0. A comparison with a sphere of the same ka and index (Fig. 1) shows that the scattered intensity for the ellipsoid is lower than the former in the backward direction, an effect partially due to more absorption.

5 Conclusion

We have derived a modified version of the eikonal model valid for large absorbing spheres ($|Im(m)|ka \gtrsim 1, ka >> 1$), by imposing that the asymptotic expressions of the eikonal model amplitude in the forward and backward limits reproduce the diffractive scattering amplitude and the backscattered amplitude obtained from geometrical optics. We have shown that one has to replace the coefficient $\gamma(\theta)$ in the formula (8) by the Fresnel reflection coefficient $r_\perp(\theta)$. Numerical results show a great improvement of the eikonal formulation, especially for scattering angles greater than 40^o and a much closer agreement with the Mie theory.

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Figure Captions

Fig.1 Geometry and notations.

Fig.2 Scattering intensity for a sphere of size parameter $ka = 50$ and refractive index $m = 1.33 + i0.05$. Mie theory (solid), modified eikonal model (dashed), original eikonal model (dotted).

Fig.3 Scattering intensity for a sphere of size parameter $ka = 20$ and refractive index $m = 1.5 + i0.1$. Mie theory (solid), modified eikonal model (dashed), original eikonal model (dotted).

Fig.4 Scattering intensity for a sphere of size parameter $ka = 60$ and refractive index $m = 2.5 + i0.05$. Mie theory (solid), modified eikonal model (dashed), original eikonal model (dotted).

Fig.5 Scattering intensity for a sphere of size parameter $ka = 20$ and refractive index $m = 5.0 + i0.1$. Mie theory (solid), modified eikonal model (dashed), original eikonal model (dotted).

Fig.6 Scattering intensity for a sphere of size parameter $ka = 100$ and refractive index $m = 2.5 + i0.01$. Mie theory (solid), modified eikonal model (dashed), original eikonal model (dotted).

Fig.7 Scattering intensity for an ellipsoid of size parameter $ka = 50$, $kb = 75$ and refractive index $m = 1.33 + i0.05$. Modified eikonal model (dashed), original eikonal model (dotted).